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Beliefs and Practice Aligned with Reform: A Sixth-Grade Teacher's Response to a Student's Conjecture about Measuring Area

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ABSTRACT

Mathematics education reforms advocate that K-12 teachers should support their students as they unpack and refine mathematical ideas and make connections among concepts. Central to the teacher's role in the classroom is to help students become comfortable articulating and developing their own ideas about mathematics and taking mathematical risks. Such pedagogy requires that the teacher's own beliefs about teaching and learning mathematics encompass students as active learners who may be guided in making and refining mathematical conjectures to build their own knowledge of mathematics. Research, however, is limited in providing insight into how middle school mathematics teachers might respond effectively to students' nonstandard ideas about mathematics. This paper examines how an experienced middle school mathematics teacher draws on her own reform-aligned beliefs to respond to a student's nonstandard mathematical idea in a way that supports the student and reform goals for school mathematics. This research can inform understanding of how beliefs can support teachers' practice in reforming middle school mathematics.

Introduction

> [S]tudents' contributions may occasionally be irrelevant or lack mathematical substance. But, even when this happens, the teacher and students can derive some benefit. It can be productive for the teacher to pick up on and probe incorrect or incomplete responses. Only by examining misconceptions and errors can students deal with them appropriately (National Council of Teachers of Mathematics [NCTM], 2000, pp. 271-272).

Part of a teacher's role in the middle school mathematics classroom is to help students develop, explore, and refine their own conjectures, ideas, and solutions (NCTM, 2000). One way that teachers do this is to respond to students' nonstandard solutions or strategies in solving problems. In doing this, teachers need to balance respecting and nurturing students as mathematical risk takers and learners with guiding them toward developing appropriate and valid understandings of mathematical concepts and procedures (Ball, 1990). A growing body of research shows that underlying teachers' ability to accomplish this in the mathematics classroom are beliefs about how students learn and beliefs about teaching mathematics that are consistent with reform goals for school mathematics (NCTM, 2000; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). Such beliefs include that students should explore and experiment in mathematics, that teachers guide and facilitate learning rather than dictate it, that students and teachers navigate mathematical terrain together and are responsible for shaping their own and others' learning of mathematics (Hill, Rowan, & Ball, 2005; Rickard, 1995, 2005).

Orchestrating mathematical discourse in the classroom in effective ways that address students' learning needs and reform goals for school mathematics is complex (Ball, 1992; Hill et al., 2005; NCTM, 2000; Ross et al., 2003). Unpacking an example of how an experienced and effective middle school mathematics teacher responds to a student's nonstandard conjecture can, however, help understand how a teacher's beliefs about teaching and learning may support her practice and shape how she uses students' mathematical ideas as a springboard in her classroom. Such inquiry fits into a substantial body of research that uses case study methods to illuminate the dynamics of teaching and learning mathematics as problem solving and implementation of the NCTM standards (NCTM, 2000; Rickard, 1995, 2005).

A Profile of Betty Walker

Betty Walker (a pseudonym) is an experienced and effective teacher who consented to work with the author over an extended period, during which she was observed teaching in her classroom and participated in informal interviews to discuss her teaching and her beliefs about teaching and learning mathematics. With over 20 years of teaching experience, she has taught at various grade levels in K-8, but especially enjoys teaching middle school students and has been teaching sixth grade for 16 years. Active in professional development and eager to try new things, Betty especially enjoys teaching mathematics. She attributes her interest in mathematics to a fondness of puzzles, problem solving, the influence of her father who was an engineer, and her own success in mathematics. Betty notes that she majored in engineering when she first began college and completed a number of advanced mathematics courses, before switching to teaching. Betty teaches sixth-grade in a suburban school district in a Midwestern state. Her middle school is ethnically diverse, enrolls about 550 students in grades six, seven, and eight, and her class includes 22 students.

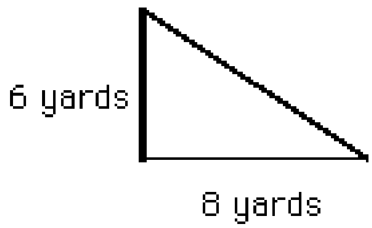
Betty Walker believes that students, particularly sixth-graders, best learn about mathematical concepts through concrete learning experiences that, typically, utilize physical objects (e.g., manipulatives such as tiles, cubes, etc.). For example, Betty described how important she thought it was for middle school students to learn about perimeter and area by physically placing square tiles on figures to measure perimeter and area. Betty argued that most curricular materials merely *describe* how area and perimeter of figures might be found by using square tiles and that

> A lot of kids at sixth-grade are not ready for this just being described. They have to have the kinesthetic, they have to pick it up, touch it, set it down, touch each tile as they count. And then after they have some time with that, to see it, then it's a piece of cake, because now they understand what perimeter and area are.

Elaborating on how she believes sixth-graders learn mathematics for understanding, Betty maintained that students must be given "a chance to discover and experiment around, and try to pull out some concepts, some patterns, looking for those kinds of things." Betty believes that this is what students have opportunities to do by using manipulatives, like tiles, to learn about perimeter and area. Betty Walker's description of how square tiles can be used to learn about perimeter and area emphasizes development of conceptual understanding. In particular, that area is the number of square units needed to exactly cover a figure and perimeter is the number of (linear) units needed to exactly surround the figure.

A Student's Conjecture about Square Units for Measuring Area

Measurement concepts (e.g., perimeter, area, surface area, volume) are among Betty's favorite mathematics topics to teach in sixth grade. During a discussion about teaching perimeter and area, the author asked Betty to respond to an example of a student's conjecture about the relationship between different kinds of square units used for measuring area. Betty studied the student's work as the author related the background of how the student explained his work and mathematical reasoning, and then Betty shared how she would respond to the student:



A = 1/2 x B x H = 1/2 x 6 x 8 = 1/2 x 48 = 24 square yards
3 feet = 1 yard so Area = 24 x 3 = 72 square feet

Figure1. Student's conjecture about square units for area.

AR: Now, suppose that during your instruction on perimeter and area one of your students raises his hand and is very excited. He says that he has figured out how to convert between different units used to measure area. And he shows you his solution to this area problem [see figure above] on this triangle and that he came up with 24 square yards. Now, he says that since there are 3 feet in a yard he can convert the area from square yards to square feet. How would you respond to this student?

Betty: Well, first of all, I'd congratulate him for putting those two things together. Then I think I would ask him if he could describe what a square foot was -- could he draw a picture for me of a square foot. If he could not I'd help him draw, you know, a square and we could measure it one foot by one foot. And then we would talk about what would be the area of that -- that's one square foot. And then I'd say, well if that works for that, does it stand then that if I take a square yard and do some the same thing, have it one yard on each of the sides, what would it's area be -- well, one square yard. And then I would ask him to look -- I might even have him cut it out. Some times if you have them cut out a square foot and then (pause) cut out a square yard, it's much easier to see that, very obviously, it's going to take more than three of these [i.e., square feet] to fill-up this [i.e., the square yard]. Well, how is that possible? Sometimes the touching, the feeling of it makes it - no matter what you would say -- that makes it so much simpler. My guess is I usually have somebody who says 'But wait a minute, you have to measure along the side and find its length first in the same units.' That means if you're going to talk about feet, well that one over there, you should say instead of being one yard long that it's three feet long on a side. Oh, well then you could say, that's like three times three -- so 1 square yard is really 9 square feet!

Betty's response to the student's work is consistent with her earlier expressed beliefs about teaching and learning mathematics. Moreover, as recommend by school mathematics reforms (e.g., NCTM, 2000), Betty sees the student's misconception about the relationship between square feet and square yards not as a mistake to be corrected but as an opportunity to explore the concept of measuring area in greater depth. The student's conjecture, though flawed, serves for Betty as an example of promising mathematical reasoning and as a vehicle for the student to further investigate, and refine his understanding of, the relationship between different square units for measuring area.

Conclusion: Beliefs and Practice Aligned with Reform

Betty Walker's expressed beliefs about teaching and learning mathematics to sixth grade students are consistent with reforms for school mathematics (NCTM, 2000). Moreover, her response shows how these beliefs translate into her thinking about teaching with students who, also consistent with reform, are active learners who propose and communicate conjectures about the mathematics they are learning. Finally, Betty's response to the student's conjecture demonstrates how she thinks about using the student's reasoning, albeit flawed, as motivation for investigating measuring area further to both refine the student's understanding and capitalize on the promise of the student's mathematical reasoning and communication.

Betty's response to the student further shows how the NCTM (2000) content standard of measurement (i.e., learning about measuring area) can be addressed simultaneously with the NCTM process standards of reasoning (i.e., developing a mathematical conjecture) and communication (i.e., explaining a mathematical conjecture). In this way, Betty's response serves as an example of how school mathematics reforms can be implemented in the middle school classroom and provides other middle school mathematics teachers with an exemplar for working with their own students to implement reforms. Studying further how teachers like Betty Walker work with their students in ways that are consistent with reforms holds promise for developing a deeper understanding of the complexities of implementing reforms, like the NCTM standards (NCTM, 2000) in school classrooms.

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