An In-Depth Analysis of the Cognitive and Metacognitive Dimensions of African American Elementary Students’ Mathematical Problem Solving Skills

Dr. Mack T. Hines III  
Sam Houston State University  
Huntsville, Texas

William Allan Kritsonis, PhD  
Professor and Faculty Mentor  
PhD Program in Educational Leadership  
The Whitlowe R. Green College of Education  
Prairie View A&M University  
Member of the Texas A&M University System  
Prairie View, Texas  
Visiting Lecturer (2005)  
Oxford Round Table  
University of Oxford, Oxford, England  
Distinguished Alumnus (2004)  
College of Education and Professional Studies  
Central Washington University

ABSTRACT

This descriptive research study analyzed the metacognitive and cognitive mathematical problem solving skills of 67 African American third and fourth grade students. The results from an administration of Desoete, De Clercq, and Roeyers’ (2000) Evaluation and Prediction assessment (EPA 2000) showed somewhat low metacognitive prediction and metacognitive evaluation skills. The students also showed lower performances on multi-sentence word problems (simple linguistic sentences (L), contextual information (C), relevant information selection (R), and mental visualization (V)) than simple sentence computational word problems (number system knowledge (K), number sense estimation (N), symbol operation (S), numerical information (NR), and procedural calculation (P)). Therefore, these students should receive needs-specific math instruction on multi-sentence word problems. From a metacognitive perspective, teachers must also develop the students’ ability to predict and reflection on chosen strategies for solving word problems. Such intensive instruction could enhance African American elementary students’ problem solving and overall mathematical skills.
Introduction

Mathematical problem solving is one of the most pivotal mathematical experiences for students (O'Connell, 2000). This concept has been called the “starting point and ending point to well balanced mathematics lessons” (O'Connell, 2000, p.37) and “site in which all of the strands of mathematics proficiency converge” (Kilpatrick, Swafford, & Findell, 2001, p. 421). Within the abundance of research on mathematical problem solving, a few studies have emerged on the mathematical problem solving skills of African American students. For example, Malloy and Jones’ (1998) study of 24 eighth grade African American students’ problem solving skills showed that they applied a holistic approach to mathematical problem solving. In addition, Moyo (2004) investigated ninth grade students’ use of graphic organizers to organize their mathematical problem solving procedures. Moyo’s findings showed that the students used the graphic organizer as a culture tool to analyze the word problems. They also used the tools to develop holistic and development view of each aspect of the mathematical word problems. Overall, the findings from this research showed that language and culture impacted minority students’ problem solving skills. However, these implications for these studies are limited to African American secondary level students. Meanwhile, African American elementary students continue to struggle with negotiating mathematics in American classrooms (Hines, 2008; Martin, 2000; National Center for Education Statistics, 2001). Evidence of their poor performance is seen in the disparity between these students’ and Caucasian American students’ performance on national and statewide mathematics assessments (See Tables 1 and 2).

Table 1: Racial Comparison of NAEP Average Mathematics Scale Scores for Fourth Grade Students and Eighth Grade Students

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Year</th>
<th>African American</th>
<th>Caucasian American</th>
<th>African American</th>
<th>Caucasian American</th>
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<td>193</td>
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<td>4</td>
<td>2007</td>
<td>222</td>
<td>248</td>
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*NAEP scale scores range from 0 to 500.
### Table 2: Racial Comparison of Pass Rate for Mathematics Portion of TAKS Assessment

<table>
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<tr>
<th>Year</th>
<th>3rd Grade Pass Rate Percentage</th>
<th>4th Grade Pass Rate Percentage</th>
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<td></td>
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<td>2000</td>
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<td>2007</td>
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In addition to these results, 31,877 (70%) of my state’s 254,539 African American elementary level students passed the math section of statewide testing (Texas Education Association, 2007). Thirty-one of the 40 test items focused on multi-sentence problem solving. Specifically, students must engage in conceptually rich thinking to construct relationships among mathematical principles, ideas, and words. Therefore, 222,662 African American third grade students struggled with 77% of the test. This outcome reflects a decade-long trend of poor African American third grade and African American fourth performances in mathematics and a national downward trajectory of African American math performance that begins in elementary school (Fryer & Levitt, 2006; Tate, 1995). These factors speak to the need to identify specific determinants of African American third grade students’ poor performance in mathematical problem solving.

One solution is to measure the specific cognitive and metacognitive aspects of their mathematical problem solving skills. Thus, the purpose of this study was to determine how African American third grade students use cognition and metacognition to make sense of mathematical word problems.

### Research Question

The research question for this study is:

1. What are the cognitive and metacognitive aspects of African American elementary level students’ mathematical problem solving skills?
Literature Review

Cognition and Metacognition

This research is centered on the role of cognition and metacognition in mathematics. Flavell (1976) defined cognition as “one’s ability to organize and execute processes in a sequential manner”. With problem solving, students translate numerical comprehension (NR), symbol comprehension (S), simple linguistic sentences (L), and contextual information (C) into a mental visualization (V) of the word problem. Next, they organize number system knowledge (K), relevant information (R) and number sense estimation (N) into a procedural calculation (P). The calculations are translated into computing the solution (Desoete, Roeyers, & Buysse, 2001).

Flavell (1976) defined Metacognition as “the ability to think about one’s thinking” (p.232). With mathematics, metacognition measures students’ predictions, monitoring, and evaluation of word problems. Prediction impacts students’ speed for working on word problems. Evaluation measures the quality of students’ reflections on strategies for achieving desired solutions. Early analyses showed that students struggle with using metacognition to make meaning of mathematical word problems (Cooney, 1985; Flavell, 1976; Schoenfield, 1989). Additionally, many teachers do not model strategies for thinking about how to validate mathematical solutions.

Empirical Research on the Analysis of Cognitive, Metacognitive Problem Solving

Desoete, De Clercq, and Roeyers (2000) organized cognitive and metacognitive problem solving processes into the Evaluation and Prediction Assessment (EPA2000). The EPA 2000 allows the teacher to obtain a fair intra-individual picture of students’ metacognitive and cognitive problem solving skills. Desoete et al. (2000) used the assessment to measure Dutch third grade students’ mathematical problem solving skills. The findings revealed that the students struggled with cognitive (L,V,C,R) and metacognitive (P, EV) problem solving. They struggled with making adequate representations and visualizations of word problems. They also lacked the skills to effectively cope with contextual information or eliminate irrelevant information. The teachers translated the results into cognitive and metacognitive profiles of each student. The profiles were used to develop individualized math instruction for the students.

My research extends the relevancy of the EPA 2000 to American elementary students. In essence, the National Council of Teachers for Mathematics’ (1999, 2000) problem solving standard specifies that all students should be able to:

- Build new mathematical knowledge through problem solving.
- Solve problems that are developed from mathematical and other contextual situations.
- Apply and develop various strategies to solve problems; and
- Reflect on the different processes needed to make sense of problem solving situations.

In other words, students must be able to engage in conceptually rich thinking by constructing relationships among mathematical numbers, ideas, facts, and symbols.

However, most elementary teachers do not possess mathematics certification. Therefore, many of these do not have a clear understanding of how to teach mathematics to students. Research has shown that elementary teachers are more likely to emphasize procedural mathematics than conceptual mathematics. In other words, they teach students how to “do” math instead of think about mathematics. As a result, teachers do not know how students think about the problems that do not have predetermined solutions. In spite of these patterns of practice, Caucasian American students continue to outperform African American students in mathematics.

Given African American elementary students’ problem-solving deficiencies (Dossey, 1993, 1994; Grouws & Cebulla, 2000), the EPA 2000 may effectively diagnose their cognitive and metacognitive skills. The findings could be translated into needs-driven math instruction for these students.

Methodology

Sample

The sample size for this study consisted of 67 African American elementary school students. The population consisted of 36 girls and 31 boys, as well as 33 third-grade students and 34 fourth-grade students. The students attended a predominantly African American elementary school in Southeast Texas.

Instrumentation

The Evaluation and Prediction Assessment (EPA2000) (Desoete, De Clerq, & Roeyers, 2000; Desoete, Roeyers, & Bussye, 2001) The TAKS mathematics benchmark tests are designed to measure students’ progress in the following areas: 1.) Number Sense; 2.) Algebra and Functions; 3.) Measurement and Geometry; 4.) Statistics, Data Analysis, and Probability; and 5.) Mathematical Reasoning. The EPA 2000 is used to measure the cognitive and metacognitive mathematical problem-solving skills of third and fourth grade students. First, children review word problems and then predict success on a 5-point rating scale. Children then perform the cognitive tasks without seeing their predictions.

Numerical comprehension and production (NR) problems encompass reading single-digit and multiple-digit numerals (Arrange from lowest to highest: 25 26 27). An example of operation symbol comprehension (S) is “Which is correct? 32+1=33 or
32x1=33”. Number system knowledge (K) focuses on comprehension of number structure (Complete this series: 37, 38,). Procedural calculation (P) measures skills in computation (34+47=).

Linguistic information (L) focuses on single-sentence language analysis (4 more than 26 is?). Contextual language (C) analyzes the ability to solve multi-sentence word problems (Bob owns two dogs. Carol owns four more dogs than Bob. How many dogs does Carol have?) Mental representation (V) measures visualization of word problems (23 is 6 less than ?). Relevant (R) word problems require students to use only relevant information (Tina has 24 dimes. Gabby has 5 dimes and 6 nickels. How many dimes do they have altogether?) Number sense (N) assesses spatial understanding of numbers (14 is nearest to? 20, 10, 33). After completing these cognitive tasks, children evaluate the performance without reviewing their calculations.

I validated the instrument through discussions with an 8-member panel of third grade teachers. The teachers confirmed the test’s measurement of third grade math skills. I then piloted the instrument on 76 African American third grade students. The findings yielded strong cronbach alphas for metacognitive prediction (.84), cognitive (.84), and metacognitive evaluation (.80) skills. Thus, this instrument has the internal consistency to measure African American third grade students’ problem solving skills.

**Results**

The students needed 1 hour and 30 0 minutes to complete the EPA2000. The students predicted (Pr) their performance on the mathematical problem solving tasks. Then they solved the mathematical problem solving tasks (NF, S, K, P, L, C, V, R, N) and evaluated (Ev) their performance. As to the prediction (Pr), they got a score of 24/40 or 60% (see Appendix).

Also the NR-tasks, the reading of single-digit exercises (9, 2, 7, 3, 4, 8, 5) was correct. The reading of multiple-digit exercises was good even when the digit name was not congruent with the number structure, with exception of the confusion of 71 and 37. The students read correctly 71, 41, 21, 40, 51, 82, 70, 91, 712, and 978. Furthermore, the students displayed good verbal numeral. They were able to discern differences between written and oral number production. The students read 52, 71, 330, 411, and 507 without mistakes. With regard to operation symbol comprehension (S-problems), the students knew the meaning of the <,>, x, and + symbols.

The number system knowledge (K-problems) was also assessed. The students could put 5 numbers (e.g. 19, 28, 37, 72, 46 or 105, 150, 35, 50, 10) in the correct order, but struggled with 12.1, 12, 16.1, 61 and as to the P-tasks, the students displayed mastery of procedural additions to be solved by mental arithmetic. But the students did struggle with double and triple digit calculations.

As to the language related word problems (L-problems), the students solved correctly 'twice 7 is ?', '1 less than 20 is?' and '1 more than 38 is?' Word problems involving an additional order factor (e.g. '?' is half of 8' and" is 2 less than 54') were
correct. C-problems or word problems based on additional context information were correctly solved in the case of the postman problem but not in the case of the baker problem, key problem and the marbles problem.

However, the students’ experienced difficulties with V-problems. They were unable to determine the answers to the following problems: “38 is 1 more than?” “17 is half of?”, “91 is 2 less than”, “48 is 1 less than?”, and “14 is twice?” With regard to relevant information problems, the students were unable to identify irrelevant information in some problems. In addition, the students struggled with number sense (N-problems). Word problems based on number sense (N-problems) were correct in the case of the flyer problem, but not in the case of the car problem, 27 near?, 99 near? and in the case of the bus problem (see 1/5 or 20% number sense Appendix). The students often misjudged their own results and got a score of 20/40 or 50% on evaluation (Ev).

In sum, the students’ cognitive strengths were numerical comprehension and production (NR), symbol comprehension and production (S), number structure (K), and linguistic information (L). Their cognitive weaknesses were contextual information (C), mental representation of the answer through visualization (V), selecting relevant information (R) and estimating in number sense tasks (N). As to the off-line metacognitive skills, we found the students retarded on prediction (Pr) skills but even more retarded on evaluation by skills. Therefore, the students should receive comprehensive cognitive strategy instruction in coping with contextual cues (C), in problem representation strategies or visualization (V), in selecting relevant information (R) and in dealing with number sense (N). Furthermore, the students should experience reflection moments after the mathematical problem solving. This step could increase their prediction (Pr) and evaluation skills (Ev).

**Concluding Remarks**

This research focused on the cognitive and metacognitive aspects of African American elementary students’ mathematical problem solving skills. The findings showed that the students have mathematical strengths and weaknesses. In addition, teachers could use these findings to develop an intra-individual picture of the cognitive processes involved in mathematical problem solving of these students. They could then develop needs-appropriate strategies for improving their mathematical problem solving skills.

**References**


Appendix A

EPA 2000

Metacognitive Prediction

Directions: Read the problems (See the Test: Part II) and then use the scale to rate your feelings about the item.

1=I am absolutely sure that I will get the wrong answer.
2=I am not really sure that I will get the wrong answer.
3=I am somewhat sure that I will get the right answer.
4=I am sure that I will get the right answer.
5=I am absolutely sure that I will get the right answer.

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Test: Part II
EPA 2000
Cognition

Directions: Solve each word problem. Show your work on the blank pieces of paper.

1. Carlos ate 6 cookies, 3 apples, and 8 doughnuts. Dina ate 4 apples. What is the total number of apples that were eaten? _____

2. 16 more than 2 is_________

3. Put these numbers in order
   31   39   35   32   _____   _____   _____   _____

4. Which is correct?
   A. 4-2=2   B. 4x2=2   C. 4+2=2   _____

5. Complete this series
   64   63   62   61   _____   _____   _____

6. 41 is farthest from_____
   A. 40   B. 45   C. 36   D. 72   _____

7. 26-1= _____

8. Pedro walks 4 blocks to school. Ned walks 2 more blocks than Pedro and 3 fewer blocks that Ralph. How many blocks does Ned walk to school? _____

9. 2 is nearest to
   A. 10   B. 5   C. 0   D. 17   _____

10. 13 is 9 more than _____

11. 18 fewer than 19 is _____

12. Jenna has 7 days left to pick 49 blueberries. She has already picked 14 blackberries. How many blackberries must Jenna pick for each of the 7 days? _____

13. 8 more than 29 is _____

14. 24 is 6 less than _____

15. Leslie jumped 12 feet from the porch. Marla jumped 6 fewer feet than Leslie. How far did Marla Jump? _____
16. What belongs in the blanks?
    99  100 ____  ____  ____

17. Sam has 3 bottles, and Amy has 4 bottles. Carlos has 3 more bottles than Sam. Lena has 2 bottles less than Amy. How many bottles does Lena have? ____

18. Arrange in the order from highest to lowest
    12  26  18  9  32  14 ____  ____  ____  ____  ____  ____

19. Choose the correct answer
    A. 4+5=20   B. 4/5=20  C. 4 X 5=20  ____

20. 3 less than 16 is ____

21. 77 is closest to
    A. 8  B. 32  C. 84  D. 100  ____

22. Sammy ate 2 pieces of chicken for lunch. He ate 5 more pieces of chicken for dinner than he did for lunch. How many pieces of chicken did he eat for dinner? ____

23. 12 is 11 more than ____

24. 9X2= ____

25. Bob has 12 keys. Ernie has 6 more keys than Bob. How many keys does Ernie have? ____


27. Look at Mario’s Work Schedule

<table>
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<tr>
<th>Day of the Week</th>
<th>Hours Worked</th>
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<tbody>
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<td>Monday</td>
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<td>Tuesday</td>
<td>6 Hours</td>
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<td>Wednesday</td>
<td>5 Hours</td>
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<td>Thursday</td>
<td>4 Hours</td>
</tr>
<tr>
<td>Friday</td>
<td>2 Hours</td>
</tr>
</tbody>
</table>

What is the total number of hours that Mario worked on the last two days of the week? ____

28. Put these numbers in order from the lowest to highest.
    12  12.1  10.1  13  11  9.1  10.1  ____  ____  ____  ____  ____  ____  ____  ____
29. Which is correct?
   A. 36-7=29  B. 36+7=29  C. 36 × 7=29

30. Fill in the blanks
   51  53  55  57  _____  _____  _____

31. Place the numbers in the order from highest to lowest
   21  13  57  _____  _____  _____

32. 9 × 9=

33. 64 divided by 8

34. 22+39=

35. 69+52

36. 20 is half of _____

37. _____ is half of 18

38. Bob is 8 years old. Bob’s brother, Chuck, is 3 years older than he. Bob’s cousin, Jim, is twice his age. How old is Jim?

39. Twice 7 is _____

40. 100 is 4 more than _____
EPA 2000
Metacognitive Evaluation

Directions: Read the problems of the new test (See the Test: Part II) and then use the scale to rate your feelings about the item.

1=I am absolutely sure that I gave the wrong answer.
2=I am not really sure that I gave the wrong answer.
3=I am somewhat sure that I gave the right answer.
4=I am sure that I will gave the right answer.
5=I am absolutely sure that I gave the right answer.

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